NTK/KW/15/5823

Bachelor of Science (B.Sc.) Semester—III Examination

MATHEMATICS

(M_5–Advanced Calculus, Sequence & Series)

Paper—V

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—1

1. (A) By using Lagrange's Mean Value Theorem, show that

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}, \; 0 < u < v$$

and deduce that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$ 6

(B) By using $\in - \delta$ technique prove that :

$$\lim_{(x,\,y)\to(1,\,1)} (x^2 + 2y) = 3.$$ 6

OR

UNIT—III

3. (A) Let $<x_n>$, $<y_n>$ be two sequences such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, where x and y are finite numbers, then prove that :

$$\lim_{n\to\infty}(x_n - y_n) = x - y.$$ 6

(B) Show that the sequence $<x_n>$, where $x_n = \frac{2^n}{n!}$ is a monotonic decreasing sequence. Also show that it is bounded and $\lim_{n\to\infty} x_n = 0$. 6

OR

(C) Show by applying Cauchy's convergence criterion that the sequence $<x_n>$ given by

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2n-1}$$ diverges.

Further show that it is monotonic increasing. 6

(D) Prove that the sequence $<x_n>$ converges if and only if it is a Cauchy sequence. 6

UNIT—IV

4. (A) Test the convergence of the series whose n^{th} term is $\left[\frac{1}{n} - \log\left(\frac{n+1}{n}\right)\right]$ by using comparison test. 6

(B) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \le 1$ by using Cauchy's Integral Test. 6

OR

(C) Test for convergence of the series

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \ldots + \frac{(n+1)x^n}{n^3} + \ldots$$

by ratio test. Also test the convergence for x = 1. 6

(D) Test the alternating series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ for convergence. Also show that it is conditionally convergent. 6

Question—V

5. (A) Find 'c' so that $f'(c) = \frac{f(b) - f(a)}{b - a}$ for $f(x) = e^x$, a = 0, b = 1. 1½

(B) Show that $\lim_{x\to 0}\lim_{y\to 0} f(x, y) = \lim_{y\to 0}\lim_{x\to 0} f(x, y)$, where $f(x, y) = \frac{x^2 + y^2}{x + y}$. 1½

(C) Find the stationary points of $u = x^2 - 4xy + 2y^2 + 2x$. 1½

(C) Investigate the continuity of function :

$$f(x, y) = \begin{cases} \dfrac{xy^2}{x^2 + y^4}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \text{ at } (0, 0) \end{cases}.$$

6

(D) Expand $f(x, y) = x^2 + xy + y^2$ in powers of $(x - 2)$ and $(y - 3)$ by using Taylor's theorem. 6

UNIT—II

2. (A) Show that the envelope of the straight line $x \cos\alpha + y \sin\alpha = \ell \sin\alpha \cos\alpha$, where α is the parameter, is the curve $x^{2/3} + y^{2/3} = \ell^{2/3}$. 6

(B) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ when $a^m b^n = c^{m+n}$, where a and b are the parameters and c is a constant. 6

OR

(C) Show that minimum value of :

$$u = xy + \left(\frac{a^3}{x}\right) + \left(\frac{a^3}{y}\right) \text{ is } 3a^2.$$

6

(D) By using Lagrange's multiplier method, find the minimum value of $x^2 + y^2 + z^2$ subject to condition $x + 2y - 4z = 5$. 6

(D) If A, B, C are the functions of x and y, then show that the envelop of $Am^2 + Bm + C = 0$ is $B^2 = 4AC$, where m is the parameter. 1½

(E) Show that the sequence $\langle \frac{n}{n+1} \rangle$ is bounded, $\forall n \in N$. 1½

(F) Find $n_0 \in N$ such that $\left| \frac{2n}{n+3} - 2 \right| < \frac{1}{5} \; \forall n > n_0$. 1½

(G) Test for convergence of $\sum 1/n^n$ by root test. 1½

(H) Test the absolute convergence of

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots$$

1½